LAPORAN AKHIR RISET

Program Penelitian Luar Negeri

MODEL MATEMATIKA PENULARAN VIRUS CORONA MENGGUNAKAN JARINGAN SARAF KOMPUTASI HEURISTIK

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UNIVERSITAS MUHAMMADIYAH BENGKULU
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I. HALAMAN IDENTITAS

1. Judul : ”Model Matematika Penularan Virus Corona Menggunakan Jaringan Saraf Komputasi Heuristik
2. Jenis Riset : Penelitian
3. Waktu Pelaksanaan : Desember 2021
4. Tim Riset
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5.3 Biaya yang disetujui oleh UMB : Rp 20.000.000,-

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II. EXECUTIVE SUMMARY

1. **TITLE OF RESEARCH** : A mathematical model of coronavirus transmission by using the heuristic computing neural networks

2. **HEAD OF RESEARCH TEAM** : Dr. Adi Asmara, M.Pd

3. **TEAM MEMBERS** : 1. Zulqurnain Sabir
   2. Sanaullah Dehraj
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5. **EXTENDED ABSTRACT** :

Introduction

The outbreak of the novel coronavirus (COVID-19) has prompted researchers worldwide to develop accurate and efficient models to predict and understand its dynamics. Mathematical models play a crucial role in simulating the spread of infectious diseases and testing possible intervention strategies. This study introduces a nonlinear model for COVID-19 transmission involving seven population classes—Susceptible, Exposed, Quarantined, Asymptomatic, Symptomatic, and Removed individuals—using a system of coupled ordinary differential equations (ODEs). The model is solved using a heuristic numerical technique based on Scaled Conjugate Gradient Neural Networks (SCGNNs).

Methodology

The model consists of a nonlinear ODE system formulated to describe the interaction and progression between population classes. The SCGNN approach, a neural network-based solver, was employed for its robustness in handling nonlinear problems. The model was tested with three different initial condition scenarios while keeping parameters constant.

To evaluate the performance of the SCGNNs, the dataset was split into:

- 70% for training,
- 16% for testing, and
- 14% for validation.

The model's effectiveness was assessed via Mean Squared Error (MSE), error histograms, regression analysis, and comparison with benchmark solutions (Runge-Kutta method).

Results

Numerical simulations for the three scenarios yielded highly accurate predictions with minimal absolute errors (AE), as visualized in the figures below:

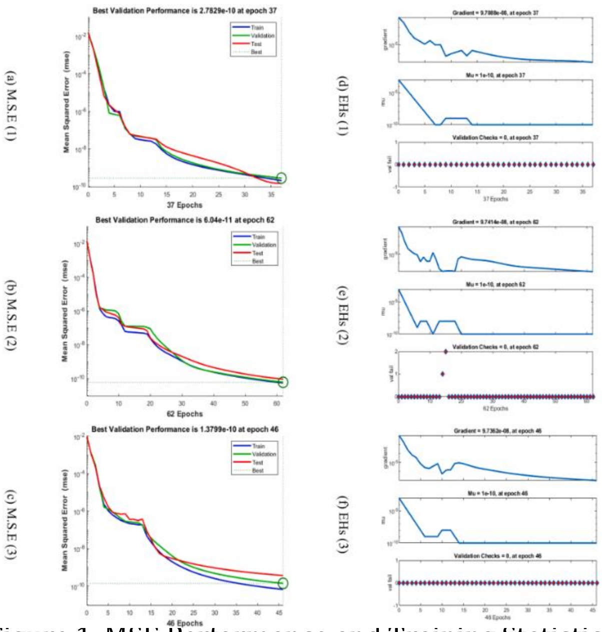

Figure 1. MSE Performance and Training Statistics

- MSE Values (Case I–III): 2.78×10^{-10}, 6.04×10^{-11}, 1.37×10^{-10}
- Training epochs: 37, 62, and 46 respectively.

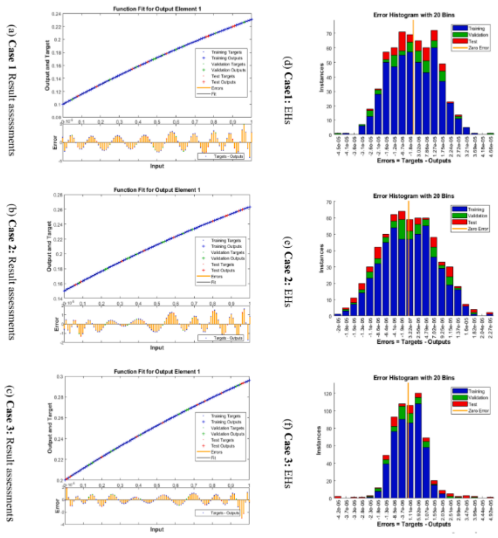

Figure 2. Regression and Error Histograms

- Absolute Errors ranged between 10^{-4} and 10^{-8} depending on the class and case.

These figures confirm high correlation and minimal prediction errors across all population classes.

Discussion

The SCGNN approach demonstrated exceptional performance in solving the COVID-19 nonlinear model. Compared to traditional numerical solvers, it offered fast convergence, reliable accuracy, and was adaptable to various initial conditions. The minimal absolute errors indicate that the model can be confidently used for forecasting and strategic planning.

Moreover, the model captures essential epidemiological behaviors by accounting for both asymptomatic and symptomatic transmissions. This enhances its applicability in public health policy modeling.

Conclusion

This study successfully models the COVID-19 transmission dynamics using SCGNNs. The approach provides high accuracy, reliability, and computational efficiency in solving complex nonlinear systems. The method is validated against known numerical techniques and shows potential for broader applications, including fractional-order models or other infectious diseases.

6. LIST OF RESEARCH OUTPUT

III. EVALUASI DIRI

1. **CAPAIAN**:
    a. TUJUAN YANG TERTULIS DI PROPOSAL: Mengembangkan dan menganalisis model matematika nonlinier untuk penyebaran COVID-19 dengan menggunakan metode Scaled Conjugate Gradient Neural Networks (SCGNNs), serta mengevaluasi tingkat akurasi, efisiensi, dan potensi aplikasinya sebagai metode prediksi numerik dalam dinamika pandemi
    b. TUJUAN YANG TELAH DICAPAI: Penelitian ini berhasil menyusun model COVID-19 dan menganalisisnya menggunakan SCGNNs, telah berhasil dicapai sepenuhnya, dan dibuktikan melalui simulasi numerik serta validasi hasil secara grafik dan statistik.
    c. TUJUAN YANG BELUM DICAPAI: validasi terhadap data nyata, penerapan dalam model lanjutan seperti fractional COVID-19 model, dan implementasi praktis dalam sistem atau kebijakan publik.
2. **ALAMAT HOMEPAGE KK YANG BERISI HASIL PENELITIAN**:
    https://www.sciencedirect.com/science/article/pii/S0955799722003976

3. **KEGIATAN DISEMINASI HASIL RISET:** *Seminar*

4. **SINERGI DENGAN KEGIATAN DAN PROYEK RISET LAIN**:

5. **KEMANFAATAN PROYEK RISET**: Yang terlibat dalam penelitian ini yaitu 8 dosen (Dr. Adi Asmara, M.Pd, Zulqurnain Sabir, Sanaullah Dehraj, Muhammad Asif Zahoor Raja, Gilder Cieza Altamirano, Soheil Salahshourf , R. Sadat, Mohamed R. Ali) dan 2 mahasiswa (Anjas Agus Prasetyo, Sania Febdana), *ada 6 kali rujukan yang bersumber pada riset ini.*

6. **PERMASALAHAN YANG DIHADAPI DAN SARAN PERBAIKAN**: Dalam pelaksanaan penelitian ini, terdapat beberapa permasalahan yang dihadapi. Salah satu permasalahan utama adalah model belum divalidasi menggunakan data riil dari penyebaran COVID-19, sehingga hasil yang diperoleh masih sebatas simulasi numerik berdasarkan parameter yang diasumsikan. Selain itu, meskipun metode SCGNNs terbukti sangat akurat dalam menyelesaikan model matematika nonlinier ini, belum ada penerapan model ke dalam platform prediksi nyata atau integrasi ke dalam sistem pendukung keputusan bagi lembaga kesehatan atau pemerintah. Permasalahan lainnya adalah model belum diuji untuk kasus turunan seperti model turunan orde pecahan (fractional-order models) yang secara teoretis disebutkan memiliki potensi untuk dikembangkan.

    Sebagai saran perbaikan, penelitian selanjutnya sebaiknya diarahkan pada validasi model terhadap data kasus COVID-19 yang nyata untuk menilai keakuratan prediksi dalam konteks dunia nyata. Selain itu, pengembangan model ke dalam bentuk fractional-order atau hybrid model akan memperluas cakupan dan ketepatan simulasi. Pengembangan aplikasi atau integrasi ke dalam sistem informasi berbasis AI juga perlu dilakukan agar hasil penelitian tidak hanya bersifat teoretis, tetapi juga memiliki nilai praktis dalam mendukung pengambilan kebijakan dan pengendalian wabah di masa depan.

7. **RENCANA KELANJUTAN PENELITIAN**: Penelitian selanjutnya akan memvalidasi model dengan data nyata, mengembangkan model fractional-order, dan menerapkan metode SCGNNs pada penyakit lain. Hasilnya juga direncanakan untuk diintegrasikan ke dalam sistem prediksi berbasis AI yang mendukung kebijakan kesehatan.

8. **TABEL ISIAN LUARAN:**

Tabel Isian Luaran

1. Publikasi di Jurnal

Tahun	Jenis Jurnal*	Judul Artikel	Nama Jurnal	P-ISSN	E-ISSN	Vol	No	Halaman (… sd….)		URL	Nama Seluruh Author	NIP Penulis Dosen	Nama Penulis Dosen	Co-Author	Nama File PDF Artikel** (dilampirkan)
2023	Jurnal Internasional	A mathematical model of coronavirus transmission by using the heuristic computing neural networks	Engineering Analysis with Boundary Elements			146		473	482	https://www.sciencedirect.com/science/article/pii/S0955799722003976	Dr. Adi Asmara, M.Pd, Zulqurnain Sabir, Sanaullah Dehraj, Muhammad Asif Zahoor Raja, Gilder Cieza Altamirano, Soheil Salahshourf , R. Sadat, Mohamed R. Ali, Anjas Agus Prasetyo, Sania Febdana	0015036508	Dr. Adi Asmara, M.Pd, Zulqurnain Sabir, Sanaullah Dehraj, Muhammad Asif Zahoor Raja, Gilder Cieza Altamirano, Soheil Salahshourf , R. Sadat, Mohamed R. Ali		https://www.sciencedirect.com/science/article/pii/S0955799722003976

*Jenis Jurnal: Jurnal Internasional;;Jurnal Nasional Terakreditasi;Jurnal Nasional Tidak Terakreditasi (Mempunyai ISSN)

**Bukti: Scan/PDF halaman pertama artikel yang memuat nama jurnal, volume, tahun, judul artikel, nama penulis, dan abstrak

2. Pemakalah Forum Ilmiah

Tahun Kegiatan	Tingkat Forum Ilmiah*	NIP Pemakalah Dosen	Nama Pemakalah Dosen	Nama Seluruh Penulis	Judul Makalah	Nama Forum	Institusi Penyelenggara	Waktu Pelaksanaan (… s.d…)		Tempat Pelaksanaan	ISBN	Status	Nama File PDF Artikel*

*Tingkat Forum Ilmiah: Tingkat Internasional;Tingkat Nasional;Regional

**Bukti: Scan/PDF halaman pertama artikel yang memuat nama forum ilmiah, judul artikel, nama penulis, dan abstrak

3. Publikasi di Media Massa

Tahun Publikasi	Tanggal Publikasi	Judul Publikasi	Jenis Media*	Nama Media	Volume	Nomor	Halaman	URL	NIP Penulis Dosen	Nama Penulis Dosen	Nama File Dokumen Pendukung (dilampirkan)**

*Jenis Media: Koran;Majalah;Tabloid;Radio;Televisi;Media Online

**Bukti: Scan artikel

4. Penyelenggaraan Forum Ilmiah

Nama Kegiatan	Unit Pelaksana	Mitra/Sponsorship	Skala Forum Ilmiah*	Waktu Pelaksanaan (…… s.d. ……)	Tempat Pelaksanaan

*Skala: Nasional; International; Regional

5. Hak Kekayaan Intelektual

NIP Dosen	Nama Inventor Dosen	Tahun Terdaftar/Granted	Judul HKI	Jenis HKI*	No. Pendaftaran	Status*	ID PATEN GRANTED	Nama File Dokumen Pendukung (dilampirkan)**

*Jenis HKI: Paten;Paten Sederhana;Hak Cipta;Merek Dagang;Rahasia Dagang;Desain Produk Industri;Indikasi Geografis;Perlindungan Varietas Tanaman;Perlindungan Topografi Sirkuit Terpadu
*Status Pendaftaran: Terdaftar;Granted
**Bukti: Halaman pertama Surat Keterangan Pendaftaran HKI atau Surat Keputusan Granted

6. Luaran Lainnya

NIP Dosen	Nama Peneliti Dosen	Tahun Kegiatan	Jenis Luaran*	Nama Luaran	Deskripsi Singkat	Nama File Dokumen Pendukung (dilampirkan)**

*Jenis Luaran: Teknologi Tepat Guna; Model; Purwarpa (Prototype); Karya Seni/Desain Kriya/Bangunan dan Arsitektur; Rekayasa Sosial
**Bukti: Berupa foto (jika ada) beserta spesifikasi teknis dari luaran

7. Buku

NIP Dosen	Nama Penulis Dosen	Tahun Penerbitan	Jenis Buku*	Judul Buku	ISBN	Jumlah Halaman	Penerbit	Nama File Dokumen Pendukung (dilampirkan)**

*Jenis Buku: Buku Ajar;Buku Teks;Modul;Panduan Praktis;Buku Profil Daerah;Buku Profil Usaha;Katalog Kegiatan Seni;Novel;Kumpulan Puisi atau Cerpen;Buku Saku;Kamus;Monograf;Biografi;Tafsir;Atas;Ensiklopedia;Lain-Lain
**Bukti: Scan cover buku

IV. LAMPIRAN *OUTPUT* PENELITIAN

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A mathematical model of coronavirus transmission by using the heuristic computing neural networks

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A R T I C L E I N F O

Keywords:
Nonlinear
COVID-19
Quarantine
Scaled conjugate gradient
SCGNNs
Numerical results
Runge-Kutta scheme

A B S T R A C T

In this study, the nonlinear mathematical model of COVID-19 is investigated by stochastic solver using the scaled conjugate gradient neural networks (SCGNNs). The nonlinear mathematical model of COVID-19 is represented by coupled system of ordinary differential equations and is studied for three different cases of initial conditions with suitable parametric values. This model is studied subject to seven class of human population $N(t)$ and individuals are categorized as: susceptible $S(t)$, exposed $E(t)$, quarantined $Q(t)$, asymptotically diseased $I_A(t)$, symptomatic diseased $I_S(t)$ and finally the persons removed from COVID-19 and are denoted by $R(t)$. The stochastic numerical computing SCGNNs approach will be used to examine the numerical performance of nonlinear mathematical model of COVID-19. The stochastic SCGNNs approach is based on three factors by using procedure of verification, sample statistics, testing and training. For this purpose, large portion of data is considered, i.e., 70%, 16%, 14% for training, testing and validation, respectively. The efficiency, reliability and authenticity of stochastic numerical SCGNNs approach are analysed graphically in terms of error histograms, mean square error, correlation, regression and finally further endorsed by graphical illustrations for absolute errors in the range of 10^{-05} to 10^{-07} for each scenario of the system model.

1. Introduction

The humanity always encountered numerous challenges and difficulties in different forms like, earthquakes, destructive floods, climate changes, infectious diseases, world wars, which seriously affected the people's lives and civilizations. Among these challenges the infectious diseases remain always huge challenge for humanity, which seriously affected the economies of many countries, education sector and tourism industry. History is full of such challenging diseases like fever caused by dengue virus [1]. The dengue fever widely spread in many countries and affected many people around globe. Every year many people admitted in hospitals due to dengue fever and most common symptoms of dengue virus include high fever, pain in joints and muscles, headache and vomiting. The HIV/AIDS [2] is another infectious disease whose first case was reported in 1976 at Democratic Republic of the Congo. Currently more than 31 million people around the globe are existing with HIV. Currently due to the awareness and developments of new treatments now HIV is somehow controllable disease. Some other deadly diseases in human history include Ebola, Lassa, Cholera, Influenza, Malaria, Smallpox, Bubonic plague [3].

Recently the world is facing another deadly novel virus known as Coronavirus. A novel Coronavirus is high transmittable and very dangerous virus, whose first case was reported in Wuhan city of China in December 2019 and rapidly spread around the globe [4]. This novel virus has badly affected routine life of the people and slows down the global economy [5–7]. The very large number of peoples died due to

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coronavirus disease and so for many variants of this virus has been discovered. The most common symptoms of COVID-19 contain dry cough, headaches, fever, sore throat and runny nose, shortness of breath. The coronavirus transmitted from people to people due to close contact and spreads rapidly which create degree of fear, worry and concern for every people, especially those having medical problem like cardiovascular disease, respiratory problems, cancer, diabetes and people of older age. The COVID-19 also creates mental and neurological complications such as stroke, agitation and delirium [8–10] among many people. The effective measures are urgently needed to control the further spread of this virus otherwise many problems like poverty, shortage of food, human loss and unemployment will further increase in future. Many countries impose lockdown and advise their peoples to wear mask, keep social distancing, avoid public gatherings, wash their hands regularly and so on [11] so that further spread of virus can be controlled. The COVID-19 badly affected the economies of many countries [12–14] and to avoid further losses proper and effective vaccine is urgently needed.

Thus, it is primary responsibility of researchers to design the mathematical models and examines the dynamics of coronavirus and presents their recommendations to control the further spread of this novel virus. In this connection many linear and nonlinear mathematical models [15–19] has been derived for novel COVID-19 to analyze it dynamics. In order to investigate the effects of different physical parameters in COVID-19 model different approaches has been used. In [20] the authors studied the COVID-19 mathematical model based on four classes namely, susceptible, healthy, infected and quarantine. The lyapunov function theory and linearization approach has been used for analyzing the stability of possible equilibrium states. In addition to these numerical simulations are also carried and found it good agreement with obtained results. The stability of SIQR epidemic model and its bifurcation value has been studied by using Routh-Hurwitz criterion in [21]. Furthermore, SIQR model also been studied by different numerical methods. It has been found that nonstandard finite difference scheme preserves all important conditions of SIQR model while others fail to preserve its conditions. In [22] authors studied non-integer order derivative mathematical model for novel COVID-19. The Fractional Euler's method has been used to obtain the numerical results of model. It has been observed that stability occurs rapidly for smaller fractional orders. In addition to this, authors also compared obtained simulations with real data of and found very close to obtained solutions. The qualitative analysis of non-integer mathematical model of COVID-19 has been presented in [23]. The uniqueness and sufficient conditions for existence of the model has been constructed by fixed point theory. Authors also used Laplace transform method together with Adomian decomposition method to construct approximate solutions and finally simulations also presented to support the obtained results. In [24] the authors studied mathematical model of novel COVID-19 via Levenberg-Marquardt artificial neural network (LMANNs) approach. The model derived in nonlinear coupled system of ordinary differential equations with different classes. The derived dataset utilized for training, validation and testing process. The reliability of design LMANNs has been achieved in terms of mean squared errors, error histograms and regression via graphical illustrations. The stochastic numerical approach has also been used to study the HIV mathematical models in [25–27]. In [28] authors used Morlet wavelet neural network (MWNN) approach to investigate second order Lane-Emden equation. In [29] authors studied fourth-order multi-singular nonlinear Emden–Fowler mathematical model by employing artificial neural network (ANNs) in combination of the global application of particle swarm optimization (PSO) enhanced by local search active set (AS) approach i.e. (ANN-PSO-AS). The effectiveness of applied approach has been verified by comparing with true results.

AI algorithm based stochastic numerical/optimization solvers have been implemented recently in different domain of applications including intelligent networks for solving the nonlinear chaotic fractional Rossler system [30], mathematical model for entropy generation in Ree-Eyring dissipative fluid flow [31], food chain models [32–33], Hall effects on MHD Jeffrey fluid flow model [34], mathematical model for the waste plastic management in ocean [35], fractional Bagley–Torvik mathematical model [36], fluidic system involving carbon nanotubes [37], thermal explosion theory [38], and stochastic numerical solution of the computer virus propagation with countermeasures model. [39]. Keeping in view of these studies of paramount significance, the aim of presented research work n this paper is to implement/investigate in AI based approach via scaled conjugate gradient neural networks (SCGNNs) for prediction of the solution for the nonlinear mathematical model of COVID-19. The SCGNNs approach has been used to compute the numerical results for proposed mathematical model. The correctness and efficiency of SCGNNs approach to investigate nonlinear mathematical models is analysed with sufficient number of graphical illustrations as well as numerical analysis.

Table 1
Physical parameter and their description [40].

Parameter	Description
Λ	Rate of Mortal recruitment
τ	Transmission rate from susceptible persons to quarantine persons
μ	Rate of natural mortality
β	Rate of interaction among susceptible and exposed persons
γ	Transmission rate of exposed to quarantine persons
η	Transmission rate between exposed to symptomatic diseased persons
σ	Rate of transfer from exposed persons to asymptomatic persons
υ	Transmission rate between quarantined to symptomatic diseased persons
θ	Transmission rate between quarantined to asymptomatic diseased persons
r_1	Rate of recovered persons from asymptomatic diseased
r_2	Rate of recovered persons from symptomatic diseased
δ	Rate of mortality in symptomatic diseased persons

This paper is arranged as follows. The nonlinear mathematical model with associated initial conditions is presented in Section 2. The proposed methodology is given in Section 3 and graphical illustrations of numerical results are also discussed in this Section. Finally, the conclusion is presented in Section 4.

2. Mathematical model

This section presents the nonlinear mathematical model of COVID-19 subject to the suitable initial conditions. The mathematical model of COVID-19 is represented by system of nonlinear ordinary differential equations and this model is based on seven classes of human population $N(t)$ and is given as: $N(t) = S(t) + E(t) + Q(t) + I_A(t) + I_S(t) + R(t)$, where $S(t)$ represents susceptible persons, the class of exposed persons is denoted by $E(t)$ and the persons quarantined due to COVID-19 are represented by $Q(t)$ whereas $I_A(t)$ represents asymptotically diseased persons and similarly symptomatic diseased persons is denoted by $I_S(t)$ and finally $R(t)$ represents the class of persons removed from COVID-19. Thus based upon these classes we have following nonlinear system of ordinary differential equations with suitable initial conditions given as under [40]:

$$\begin{cases} \dfrac{dS(t)}{dt} = \Lambda - (\tau+\mu)S(t) - \beta S(t)E(t), & S(0)\ge 0 \\ \dfrac{dE(t)}{dt} = \beta S(t)E(t) - (\gamma+\mu+\eta+\sigma)E(t), & E(0)\ge 0, \\ \dfrac{dQ(t)}{dt} = \tau S(t) + \gamma E(t) - (\mu+\upsilon+\theta)Q(t), & Q(0)\ge 0, \\ \dfrac{dI_A(t)}{dt} = \sigma E(t) + \theta Q(t) - (\mu + r_1)I_A(t), & I_A(0)\ge 0, \\ \dfrac{dI_S(t)}{dt} = \eta E(t) + \upsilon Q(t) - (\delta+\mu+r_2)I_S(t), & I_S(0)\ge 0, \\ \dfrac{dR(t)}{dt} = r_1 I_A(t) + r_2 I_S(t) - \mu R(t), & R(0)\ge 0, \end{cases} \tag{1}$$

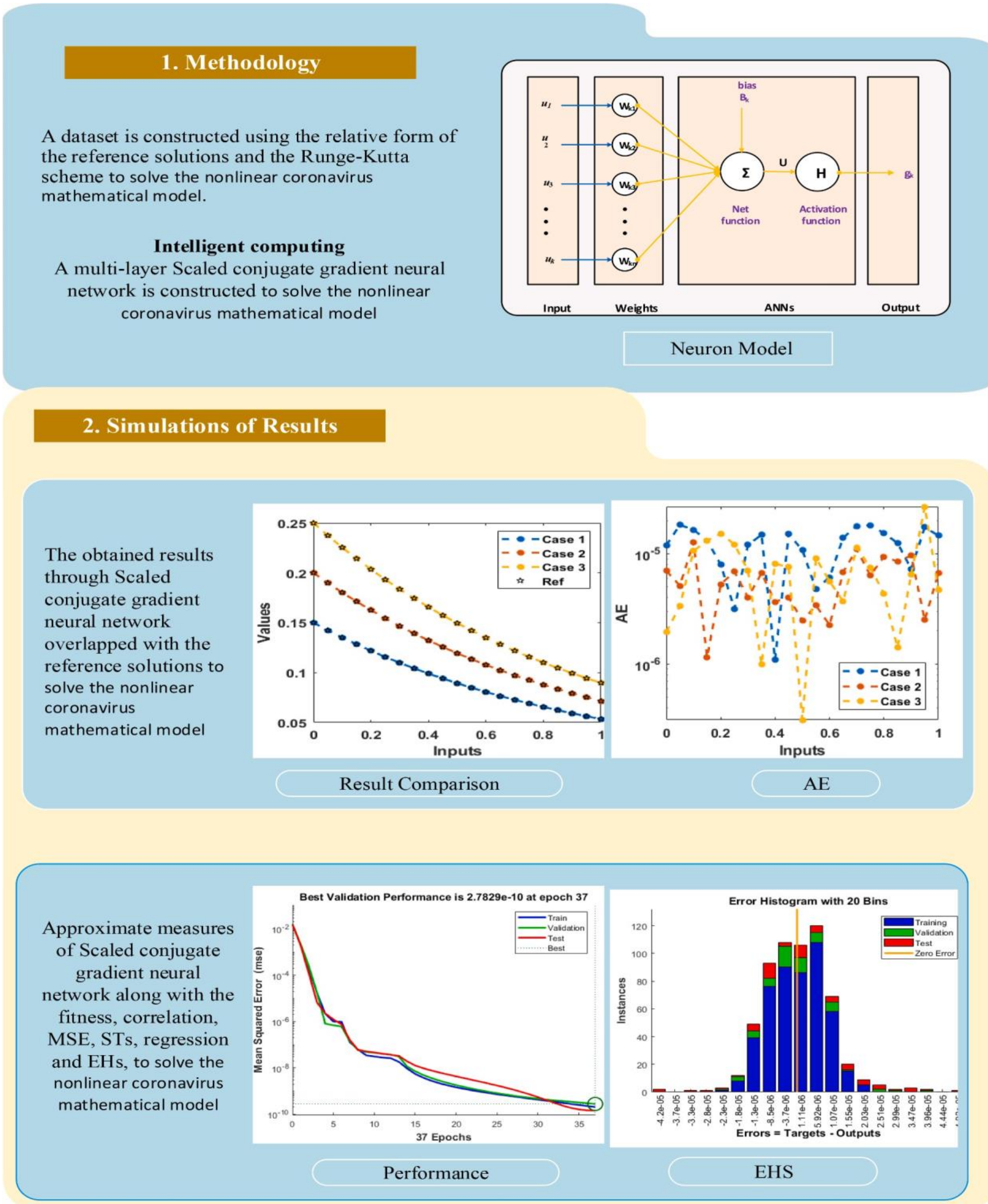

Fig. 1. Workflow illustrations of the SCGNNs to study the mathematical of COVID-19.

The physical parameters of nonlinear mathematical model of COVID-19 pandemic as portrayed in set of equations (1) are taken from reported study [40] and defined in Table 1.

We have considered the following suitable parametric values to study the nonlinear mathematical model given in Eq. (1) and are given as under:

$$\Lambda = 0.2,\ \tau = 0.1,\ \mu = 0.3,\ \beta = 0.1,\ \delta = 0.14,\ \gamma = 0.15,\ \eta = 0.25,\ \sigma = 0.35,\ \theta = 0.4,\ \upsilon = 0.12,\ r_1 = 0.35,\ r_2 = 0.45$$

The necessary justification along with theoretical convergence proof of the values of parameters in terms of global and local stability index is provided in reported [40] while the mathematical model given in Eq. (1) with above parametric values becomes as under:

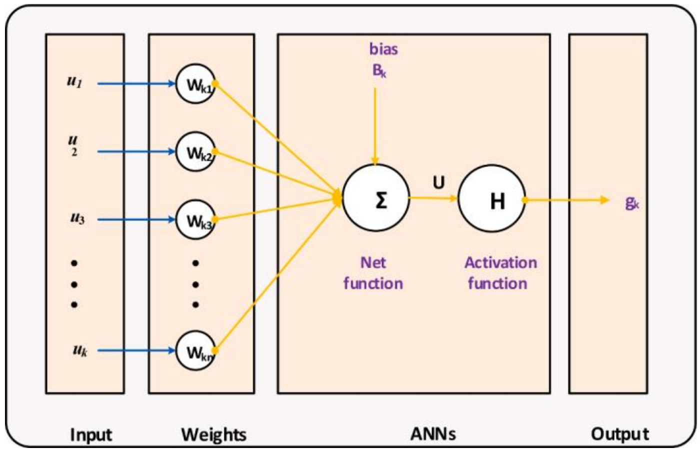

Fig. 2. The organization of single neuron.

$$
\begin{cases}
\dfrac{dS(t)}{dt} = 0.2 - (0.1 + 0.3)S(t) - 0.1S(t)E(t), & S(0) \geq 0 \\
\dfrac{dE(t)}{dt} = 0.1S(t)E(t) - (0.15 + 0.3 + 0.25 + 0.35)E(t), & E(0) \geq 0, \\
\dfrac{dQ(t)}{dt} = 0.1S(t) + 0.15E(t) - (0.3 + 0.12 + 0.4)Q(t), & Q(0) \geq 0, \\
\dfrac{dI_A(t)}{dt} = 0.35E(t) + 0.4Q(t) - (0.3 + 0.35)I_A(t), & I_A(0) \geq 0, \\
\dfrac{dI_S(t)}{dt} = 0.25E(t) + 0.12Q(t) - (0.14 + 0.3 + 0.45)I_S(t), & I_S(0) \geq 0, \\
\dfrac{dR(t)}{dt} = 0.35I_A(t) + 0.45I_S(t) - 0.3R(t), & R(0) \geq 0,
\end{cases}
\tag{2}
$$

Furthermore, The viable, robust and stable performance of proposed integrated intelligent networks via scaled conjugate gradient neural networks (SCGNNs) is analyzed in case of sundry scenarios of nonlinear mathematical model of COVID-19 pandemic as portrayed in set of Eqs. (1-2) for three different set of the initial conditions while the parameter values are remained fixed as given in reported study [40] as given below:

Case I: $S(0) = 0.1$, $E(0) = 0.15$, $Q(0) = 0.2$, $I_A(0) = 0.25$, $I_S(0) = 0.3$, $R(0) = 0.35$
Case II: $S(0) = 0.15$, $E(0) = 0.2$, $Q(0) = 0.25$, $I_A(0) = 0.3$, $I_S(0) = 0.35$, $R(0) = 0.4$
Case III: $S(0) = 0.2$, $E(0) = 0.25$, $Q(0) = 0.3$, $I_A(0) = 0.3$, $I_S(0) = 0.4$, $R(0) = 0.45$

In next section we will provide methodology to study nonlinear model subject to three different cases of initial conditions.

3. Methodology: SCGNNs

This section presents the methodology to investigate the nonlinear mathematical model of COVID-19 given in Eq. (2) subject to three different cases of initial conditions along with suitable values of different physical parameters. For investigating the nonlinear mathematical model of COVID-19 the scaled conjugate gradient neural networks (SCGNNs) approach has been used. The numerical stochastic SCGNNs approach has great capability to solve large class of (non)linear mathematical models of different fields. In this study, we used SCGNNs approach to compute the numerical performance of nonlinear mathematical model of COVID-19 based on three factors including sample statistics, testing and training. For this purpose the statics considered as 70%, 16% and 14% for training, testing and validations respectively. In addition to this, the effectiveness of numerical stochastic SCGNNs approach to investigate the nonlinear mathematical models has been analyzed graphically by using different graphs of like, error histograms, mean square error, correlation and regression and graphs for absolute errors are also drawn. Some important features of numerical stochastic SCGNNs approach to study the nonlinear mathematical models are given as under:

- The stochastic SCG approach together with artificial neural NNs has never been used before this to investigate COVID-19 nonlinear mathematical model.
- The effectiveness and correctness of the SCGNNs approach to investigate nonlinear mathematical model of COVID-19 is verified by Runge-Kutta scheme.
- The efficiency and authenticity of stochastic numerical SCGNNs approach to study the nonlinear mathematical model of COVID-19 is witnessed the through graphs of negligible absolute error (AE).
- In addition to this, the reliability of designed SCGNNs approach to deal nonlinear mathematical model of COVID-19 has also been

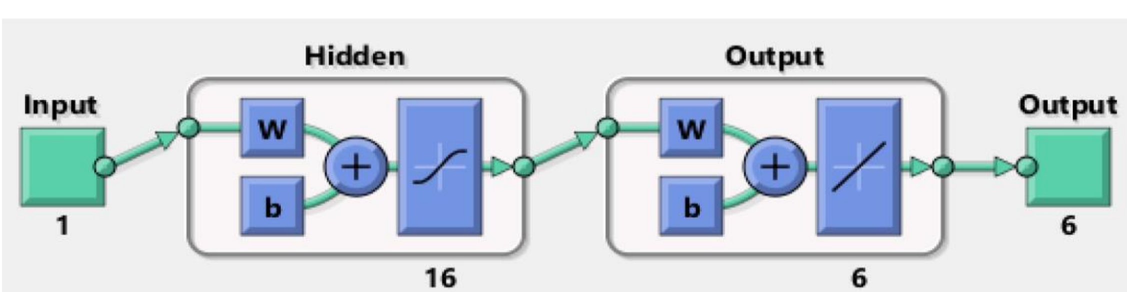

Fig. 3. SCGNNs for nonlinear system with 16 neurons.

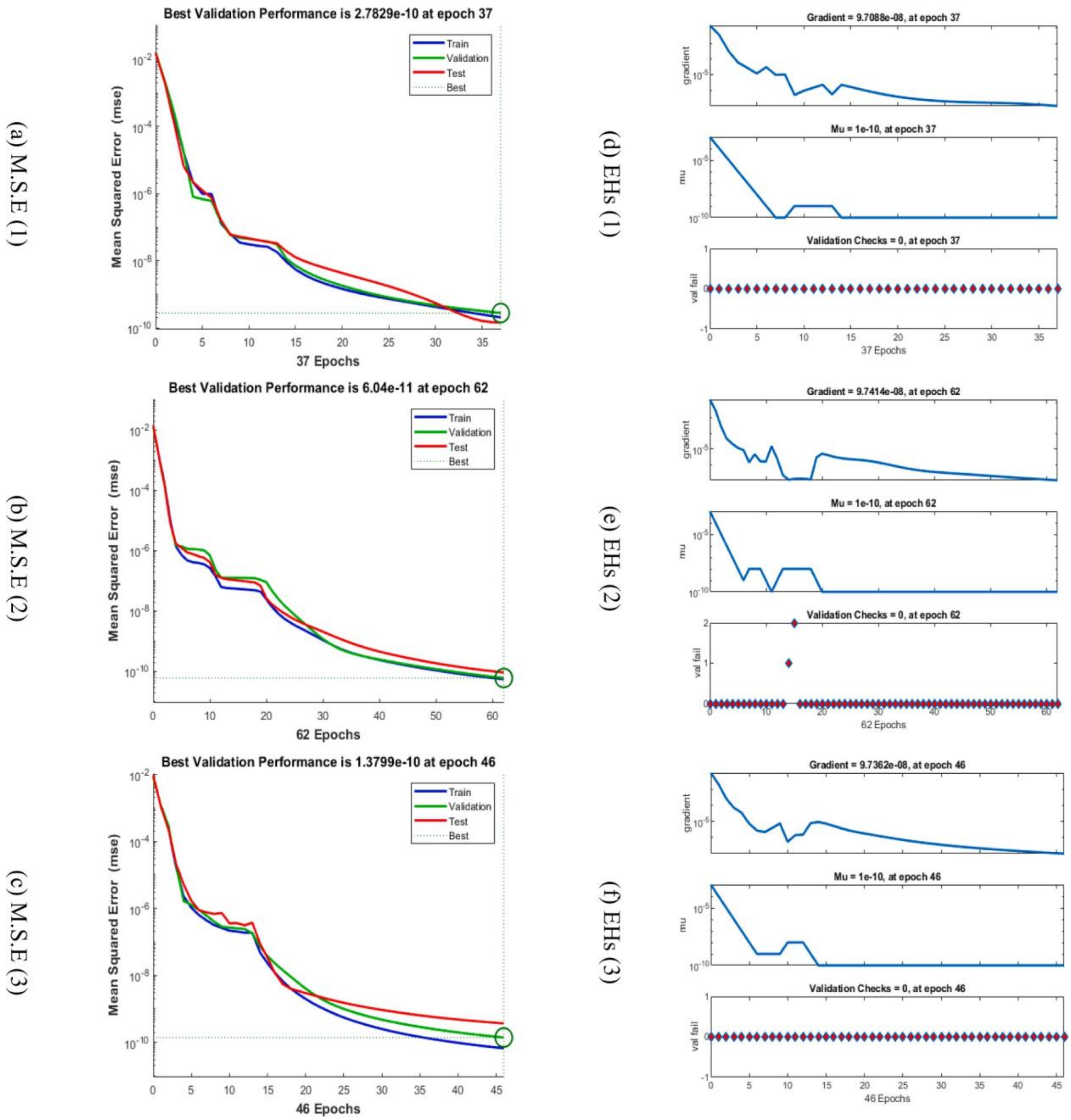

Fig. 4. MSE and STs to investigate the proposed mathematical model of COVID-19.

analyzed through the graphs of regression, EHs, correlation, MSE and STs.

- Furthermore, numerical stochastic SCGNNs approach can also be applied to investigate (non)linear mathematical models of with fractional order as well.

Fig. 1 represents an appropriate optimization flow chart with multi neuron model based on stochastic numerical SCGNNs approach to investigate the nonlinear mathematical model of COVID-19. This flow chart shows that, in first step necessary performance by using stochastic SCGNNs will be carried out, and in next step numerical simulations will be drawn. The single neuron structure is presented in Fig. 2. The statics is 70%, 16% and 14% for training, testing and validation is used for investigating the COVID-19 nonlinear mathematical model. The label data is used for training samples, i.e., inputs with prior known targets, for formulations on the networks by optimization of mean square error (MSE) based figure of merit. While for both testing and validation samples unlabelled data is used, i.e., inputs without prior knowledge of target, in order to access the performance of proposed scaled conjugate gradient neural networks fo solving nonlinear mathematical model of COVID-19 pandemic as portrayed in set of equations (1). In this work, the numerical results of proposed system are achieved by utilizing 16 neurons by using stochastic SCGNNs approach and are structured in Fig. 3.

In Fig. 3, the neural networks structure is depicted in terms of inputs, i.e., vector representing the grid of inputs time t that is independent variable of the system (1), output, i.e., matrix represented the approximate solution for 6 different classes as portrayed in system (1), hidden layers operated with 16 neurons and log-sigmoid based activation/ transfer function, backpropagation algorithm of Levenberg Marquardt is exploited for learning. Moreover, all these parameter set with care to avoid the degradation of the results due to over/underfilling of the networks.

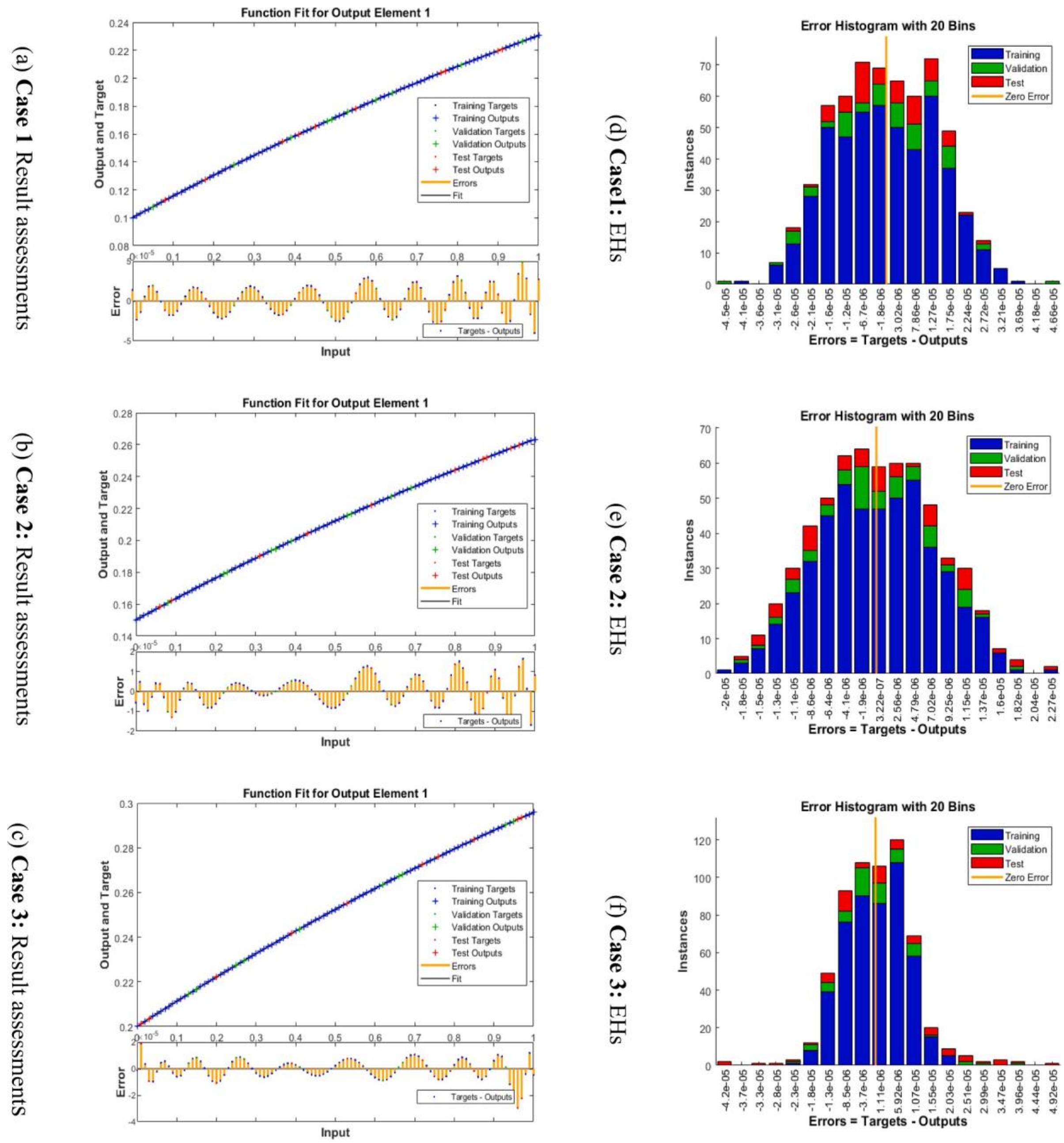

Fig. 5. Valuations and EHs to investigate the proposed mathematical model of COVID-19.

4. Numerical experimentation with interpretations

The numerical simulations to examine the proposed mathematical model of COVID-19 by using stochastic SCGNNs approach are presented in Figs. 4–7 and are illustrated for three different cases of initial conditions. Fig. 4 represents the STs and MSE performance to investigate the COVID-19 nonlinear mathematical model. The Figurers 4(a-c) depicts the illustrations of best curve, verification, training, and testing by MSE, whereas Fig. 4(d-f) represents the STs best values for solving the nonlinear mathematical model of COVID-19 at the epochs 37, 62 and 46 for three different cases of initial conditions respectively. Fig. 4(a-c) depicts that the best validation performance values for solving nonlinear COVID-19 mathematical model for three different cases are 2.7828×10^{-10}, 6.04×10^{-11} and 1.3799×10^{-10} respectively. Furthermore, Fig. 4 (d-f) represents the gradient measures values as 9.7088×10^{-08}, 9.7414×10^{-08} and 9.7362×10^{-08} for first, second and third case of respectively

Table 2
Statistical measures for the proposed model.

Case	MSE			Performance	Gradient	Mu	Epoch	Time
	[Training]	[Verification]	[Testing]					
1	2.060×10^{-10}	2.782×10^{-10}	1.44×10^{-10}	2.06×10^{-10}	9.71×10^{-08}	1.0×10^{-10}	37	1
2	5.549×10^{-11}	6.039×10^{-11}	9.18×10^{-11}	5.55×10^{-11}	9.74×10^{-08}	1.0×10^{-10}	62	1
3	6.593×10^{-11}	1.379×10^{-10}	3.63×10^{-10}	6.59×10^{-11}	9.74×10^{-08}	1.0×10^{-10}	46	1

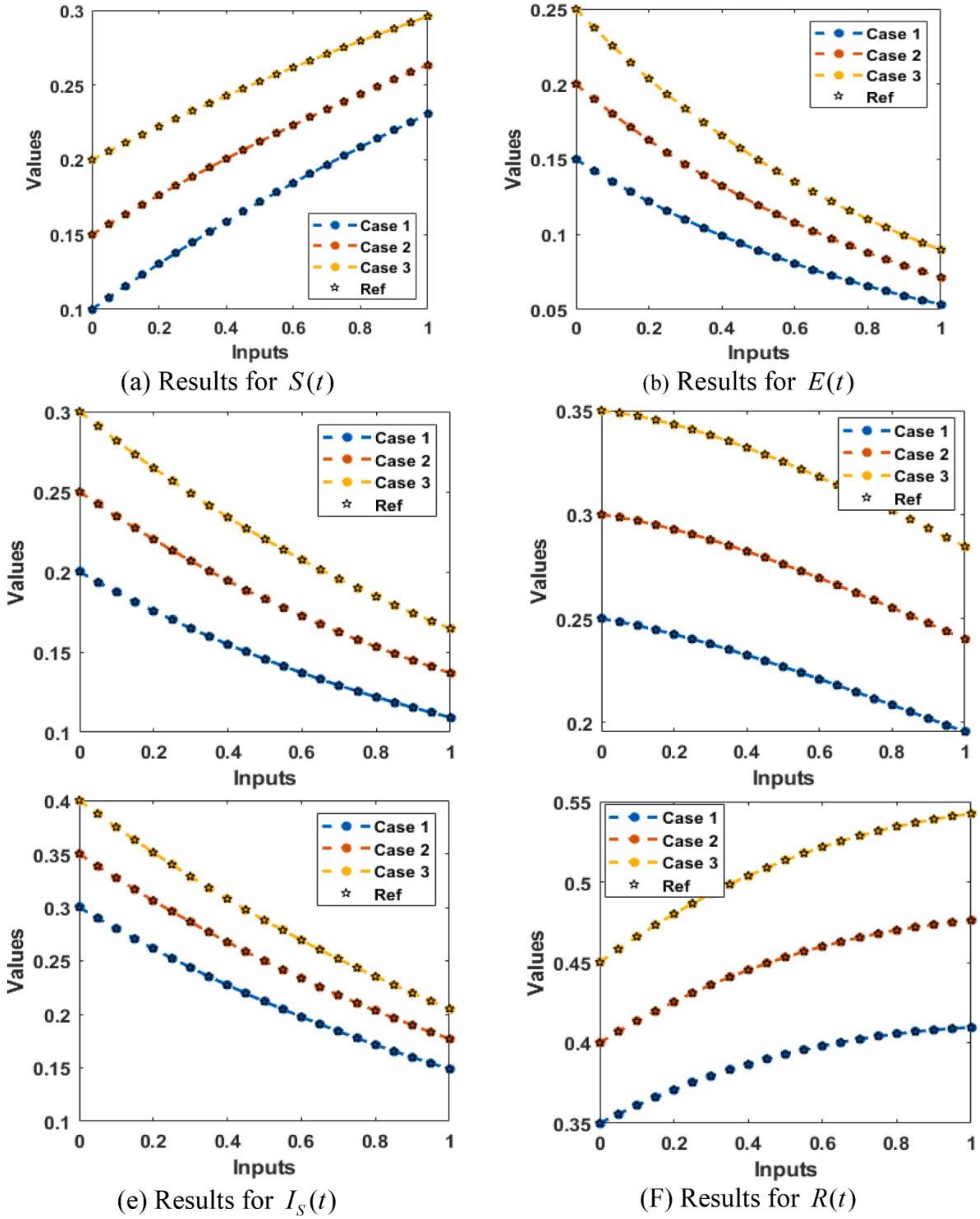

Fig. 6. Comparison of the result to investigate proposed mathematical model of COVID-19.

for solving nonlinear mathematical model of COVID-19. The efficiency and exactness for solving the nonlinear models by stochastic SCGNNs is witnessed by these obtained results.

The Fitting and error histograms EHs for solving nonlinear mathematical model by the approach of SCGNNs are illustrated in Fig. 5. Fig. 5(a-c) depicts the training, validation, test target error and fit relationship for each case of initial conditions. However, it can easily observed from Fig. 5(d-f) that EHs values i.e. zero error occurs at 1.8×10^{-06}, 3.22×10^{-07} and 1.11×10^{-06} for first, second and third case respectively for considered nonlinear mathematical model. Furthermore, the statistical values for training, verifications testing and performance for each case is presented in Table 2 as under:

Finally, the graphical illustration for comparison and absolute error AE are drawn in Figs. 6 and 7, respectively for each case to study the proposed mathematical model of COVID-19. The authenticity and efficiently of stochastic numerical SCGNNs approach to examine the nonlinear mathematical model of COVID-19 is clearly depicted in graphs drawn for AE. The graphs drawn in Fig. 6 depict the matching of proposed and reference results that justify the authenticity and exactness of designed stochastic SCGNNs approach for solving nonlinear mathematical models. In this study, nonlinear mathematical model of COVID-19 has been studied seven different classes of human population subject to three different initial conditions.

The AE for each class and each case of nonlinear mathematical model

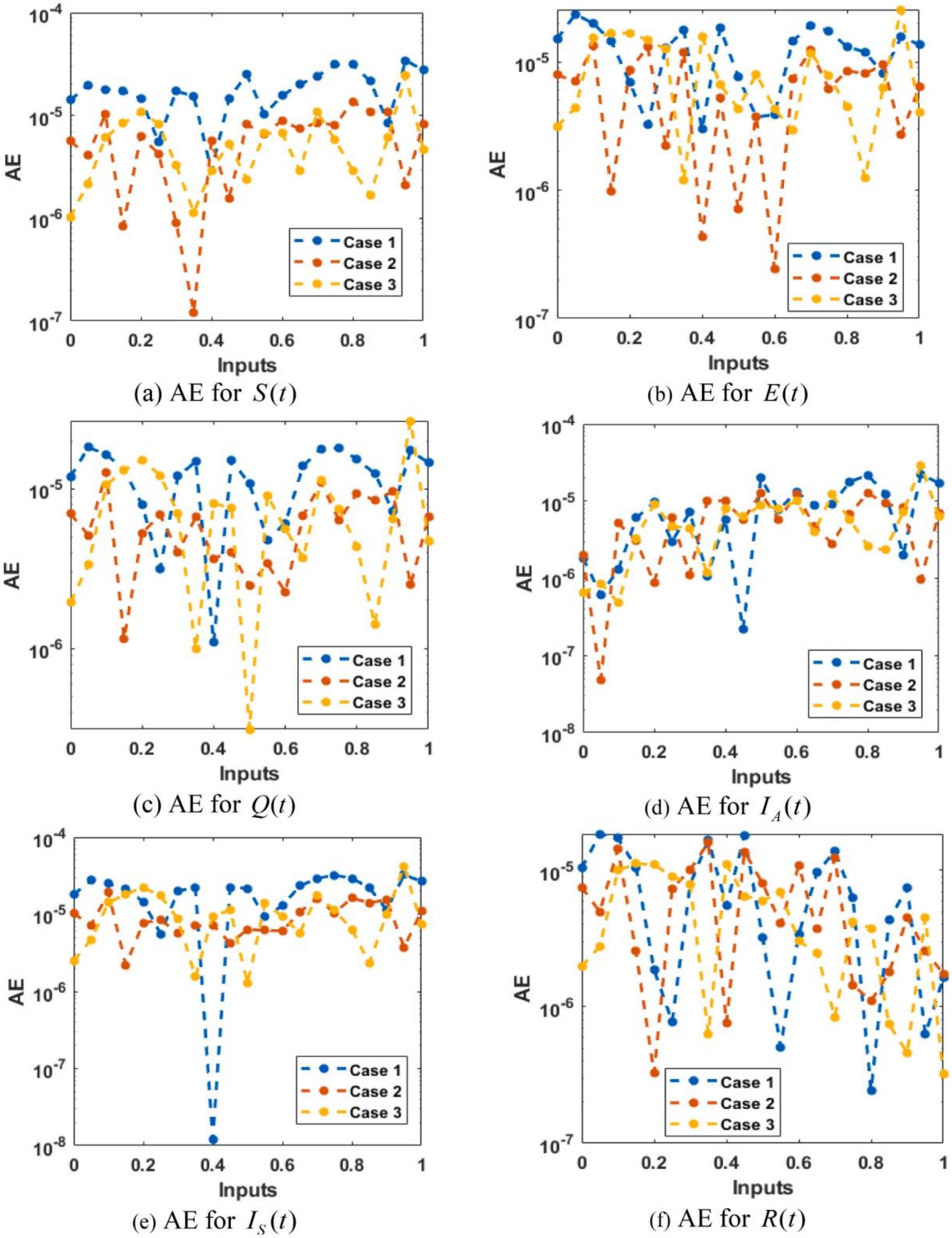

Fig. 7. AE to investigate proposed mathematical model of COVID-19.

are illustrated in Fig. 7. It can be observed from Figure 8(a) that for susceptible individuals$S(t)$ the AE lies between 10^{-04} to 10^{-05}, 10^{-05} to 10^{-07} and 10^{-05} to 10^{-06}, for first, second and third case of initial conditions respectively. Fig. 7(b) depicts the AE based exposed individuals$E(t)$ lies between 10^{-05} - 10^{-06}, 10^{-05} - 10^{-07} and 10^{-05} - 10^{-06} for first, second and third case respectively. The AE for quarantined individuals $Q(t)$ is drawn in Fig. 7(c) and it can be observed that the AE for each case lies between, 10^{-05} to 10^{-06}. Fig. 7(d) is drawn to show the AE for asymptotically infected individuals$I_A(t)$ and AE lies between, 10^{-05} to 10^{-07}, 10^{-05} to 10^{-08} and 10^{-05} to 10^{-07}, for first, second and third case respectively for nonlinear mathematical model of COVID-19. Similarly, Fig. 7(e) is drawn to depict the AE for symptomatic infected individuals$I_S(t)$ and AE values lies for 10^{-04} to 10^{-08}, 10^{-05} to 10^{-06} and 10^{-04} to 10^{-06}, for first, second and third case respectively.

Finally, it can easily be seen that the AE for removed individuals$R(t)$ lies between 10^{-05} to 10^{-07}, for each case. These all values of AE for each case for nonlinear mathematical model of COVID-19 provide the efficiency and correctness of designed stochastic SCGNNs approach.

5. Conclusions

In this paper, the nonlinear mathematical model of COVID-19 has been studied subject to seven classes of human population. The COVID-19 nonlinear mathematical model represented by system of ordinary differential equations with suitable initial conditions. This nonlinear system of ODEs has been investigated by using scaled conjugate gradient neural network (SCGNNs) and has been studied for three different cases of initial conditions with suitable values of physical parameters. Furthermore, the numerical computations has been carried out by using stochastic computing SCGNNs for verifications purpose based on sample statistics, testing and training. For this purpose, 70%, 16% and 14% data has selected for training, testing and validations respectively. The exactness and efficiency of stochastic SCGNNs approach has been validated by Runge-Kutta method. The obtained results have been illustrated graphically using error histograms, mean square error correlation and regression. The efficiency and correctness of proposed stochastic SCGNNs approach has been witnessed from numerical simulations. The ease in implementations of SCGNNs, simple concept, extendibility and applicability for still and non-stiff problems are other noticeable advantages of the proposed approach.

In future, the stochastic numerical SCGNNs approach can also be used to study the same mathematical model of COVID-19 in terms of non-integers derivative. Moreover, the proposed scheme will be implemented to solve various differential models [41–44].

Declaration of Competing Interest

All authors describe that there are no potential conflicts of interest.

Data availability

Data will be made available on request.

Acknowledgment

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RESEARCH COLLABORATION CONTRACT
UNIVERSITY OF MUHAMMADIYAH BENGKULU
WITH
HAZARA UNIVERSITY, MANSEHRA, PAKISTAN

On this Tuesday, the twentieth of December, in the year Two Thousand Twenty-Three, we are the undersigned :

PIHAK I

Name : Zulqurain Sabir
Qualification : Master's dan MPhil in Mathematics, University of Punjab & Preston University, Pakistan
Institusi : Hazara University, Mansehra, Pakistan

In this agreement, hereinafter referred to as PIHAK I

PIHAK II

Name : Dr. Adi Asmara, M.Pd.
Institution : Universitas Muhammadiyah Bengkulu

In this agreement, hereinafter referred to as PIHAK II

The two parties agreed to establish international research cooperation with the following conditions:

PASAL 1

Research title:

MATHEMATICAL MODEL OF CORONAVIRUS TRANSMISSION MENGGUNAKAN JARINGAN

The objectives of this cooperation are:

a. Conducting joint research in the development of a mathematical model of the spread of the Corona virus based on heuristic neural networks.
b. Produce research outputs in the form of international scientific articles, computational models, and supporting mathematical instruments.
c. Strengthening the academic collaboration network between Hazara University Pakistan and Muhammadiyah University Bengkulu.

PASAL 2

The scope of the research includes:

a. Preparation of mathematical model design and computational neural network structure.
b. Analysis of cases of the spread of the Corona virus based on data and mathematical simulations.
c. Exchange of academic knowledge, scientific discussion, and division of roles in research.
d. Joint publications in international journals are indexed as mandatory outputs of collaboration.
e. Seminars, workshops, and dissemination of research results at both institutions.

PASAL 3

1. The collaboration is carried out through *an in-kind research collaboration* scheme.
2. Contribution of PIHAK I (Hazara University Pakistan):
   a. Provision of expertise in mathematical modelling and neural networks.
   b. Guidance on data analysis, algorithms, and computational modeling
   c. *Academic supervision* and *peer review* support .
3. Contribution of PIHAK II (UMB):
   a. Implementation of experiments, data processing and implementation of computing.
   b. Preparation of publications, research reports and coordination of research activities.
4. There is no cash transfer during the collaboration, unless provided for in a new addendum in writing.

PASAL 4

a. Provide expertise support in algorithm design and neural heuristic computing.
b. Guiding mathematical analysis and modeling of virus spread.
c. Actively involved in manuscript writing and international publications.
d. Maintain scientific ethics and the integrity of research results.

PASAL 5

a. Carry out research according to collaborative methods and agreements.
b. Manage data, simulation, and computational implementation of transmission models.
c. Make progress reports to PARTY I and complete joint publications.
d. Maintain international academic research and ethical commitments.

PASAL 6

a. Data, mathematical models, algorithms and research findings are the joint property of both parties.
b. Publications must use a joint authorship format by listing both institutions.
c. The order in which articles are written is determined based on the scientific contributions of each party.
d. Research results cannot be published unilaterally without the consent of the collaboration partner.

PASAL 7

a. The contract is valid for 8 months from the date it is signed.
b. The extension of cooperation is allowed with the written agreement of both parties.

PASAL 8

The parties are obliged to maintain the confidentiality of research data that has not been published. This provision remains valid even if the cooperation ends.

PASAL 9

All disputes are resolved through deliberation. If no agreement is reached, the settlement is carried out based on the international academic law mechanism agreed upon by both parties

PASAL 10

The contract is drafted in two original duplicates and has the same legal force. The parties sign this document knowingly, voluntarily, and without coercion.

The Undersigned

Zulqurain Sabir **D r. Adi Asmara, M.Pd**